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Chiral Invariant Phase Space Event Generator

I. Nucleon-Antinucleon Annihilation at Rest

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Abstract. The CHIPS model and its first implementation within the GEANT4 simulation software package are considered. Hadron production in the process of nucleon-antinucleon annihilation is used as the basic example showing the structure of the model and corresponding software modeling algorithms. Model calculations of multiplicities and spectra of secondary hadrons in the annihilation process are compared with experimental data.

PACS. 02.70.Lq Monte Carlo and statistical methods – 12.38.Mh Quark gluon plasma – 13.75.Cs Nucleon-nucleon interactions

1 Introduction

The present publication is the first in a series describing the CHIPS model which we propose to use in the GEANT4 [1] simulation of hadronic processes in the intermediate energy range from about 100 MeV to approximately 10 GeV. The basic idea of the model, a hadronization of quark-partons in nuclear matter (one quark-parton into one outgoing hadron), appeared in 1984 [2], when secondary spectra of hadrons in high energy nuclear reactions (pions, protons, neutrons, deuterons, etc.) were found to be a reflection of the quark-parton spectra. The first implementation of this basic idea in the form of a computer program – Monte Carlo event generator – was attempted in 1988 - 1992 at ITEP, Moscow. The DINREG [3] code for nuclear fragmentation, i.e., production of pions, nucleons, and nuclear fragments in the inelastic interaction of different projectile particles with nuclei, has been used as a tool in calculations of detection efficiency and acceptance in the analysis of different experimental data [4,5]. In 1995 it was implemented within the GEANT3 package [6] to model medium energy photonuclear and electronuclear reactions. This implementation is extensively used at Jefferson Lab to determine radiation background in the interactions of few-GeV electrons with experimental targets.

CHIPS is a quark-level 3-dimensional event generator for the fragmentation of excited hadronic systems into hadrons. An important feature of the model is its universal phase space approach to different types of excited hadronic systems including nucleon excitations, hadron systems produced in e^+e^- interactions, high energy nuclear excitations, etc. Exclusive event generation, which models hadron production conserving energy, momentum, and charge, generally results in a good description of particle multiplicities and spectra in multihadron fragmentation processes. All this makes it possible to use the event generator in exclusive modeling of hadron cascades in materials.

We start the series with the discussion of the general features of the model and its implementation in the relatively "simple" process of multihadron fragmentation in nucleon-antinucleon annihilation.

2 Physics model

In the CHIPS model any excited hadronic system is considered to be a quasmon, a bubble containing massless quarks (quark-parton plasma). Quark-partons of the quasmon are homogeneously distributed over the invariant phase space. The quasmon can be considered as a bubble of the 3-dimensional Feynman-Wilson [7] parton gas. A quark fusion mechanism is applied to the successive fragmentation of a quasmon into hadrons. The model can be considered as a generalization of the chiral bag model of hadrons [8] in which any hadron consists of a few quark-partons. Any interaction between hadrons is modeled as purely kinematical effect of quark exchange reaction, and any quasmon decay is modeled as a quark fusion of two quark-partons in the quasmon. This approach does not pretend to be a dynamical model. It can be considered also as a generalization of the well-known hadronic phase space distribution [14] approach, because it generates not only angular and momentum distributions for a given set of hadrons but the multiplicity distribution for different kinds of hadrons too. In comparison with other parton models [9] the CHIPS model is 3-dimensional.

The invariant phase space distribution as a paradigm of thermalized chaos is applied to quarks, and simple kinematical mechanisms are used to model the hadronization of quarks into hadrons. Any quark-parton in a quasmon has a possibility to pick up another quark-parton from the same hadronic system (quark fusion mechanism) or exchange with a quark-parton of the neighboring quasmon (quark exchange mechanism). The kinematical condition for these mechanisms is that the secondary hadrons must be produced on their mass shell. In case of massless quarks the hadronization process can be easily integrated and modeled. That is why – for the sake of acceleration of the algorithm – we consider u, d, and s quarks to be massless in spite of their known nonzero mass values. Indirectly the quark mass is taken into account in the masses of outgoing hadrons. The selection of the type of the outgoing hadron is performed using combinatorial and kinematical factors which would allow given hadron to be emitted, taking into account conservation laws. In the present version of CHIPS all mesons with 3-digit PDG Monte Carlo codes [10] up to spin 4 and all baryons with 4-digit PDG codes up to spin $\frac{7}{2}$ are implemented.

The only non-kinematical concept of the model is the hypothesis of critical temperature of the quasmon, which has its almost 35-year-old history, starting with [11]. It is based on the experimental observation of regularities in the inclusive spectra of hadrons produced in different reactions at high energies. The concept of critical temperature is used in the method of calculating the number of quark-partons in a quasmon. In an infinite thermalized system the mean energy of partons is $2T$ per particle, where T is the temperature of the system. For the finite system of N partons with total center-of-mass energy M the invariant phase space integral (Φ_N) is proportional to M^{2N-4}, where according to the dimensional counting rule $2N$ comes from $\prod_{i=1}^{N} \frac{d^3p_i}{E_i}$, and 4 comes from the energy and momentum conservation $\delta^4(\mathrm{P}-\sum \mathrm{p}_i)$ function. On the other hand, at a temperature T the statistical density of states is proportional to $e^{-\frac{M}{T}}$. So the probability to find a system of N quark-partons in a state with mass M is $dW \propto M^{2N-4}e^{-\frac{M}{T}}dM$. For this kind of a probability distribution the mean value of M^2 can be calculated as

$$< M^2 >= 4N(N-1)\cdot T^2. \tag{1}$$

When N goes to infinity one can obtain for massless particles the well-known $< M > \equiv \sqrt{< M^2 >} = 2NT$ result. If a nucleon is excited, for example by absorbing energy from an incident real or virtual photon, the number of partons in the newly formed quasmon is determined by equation (1). As the number of quark-partons in the quasmon is set this way, the spectrum of quark-partons can be calculated using the same $\Phi_N \propto M^{2N-4}$ relation applied to the residual $N-1$ quarks. As a result, the quark-parton spectrum can be calculated as:

$$\frac{dW}{kdk} \propto (M_{N-1})^{2N-6}, \tag{2}$$

where M_{N-1} is an effective mass of the residual $N-1$ quark-partons. It can be calculated as a function of the total mass M:

$$M_{N-1}^2 = M^2 - 2kM, \tag{3}$$

where k is the energy of the primary quark-parton in the center-of-mass system (CMS) of N partons. The resulting equation for the quark-parton spectrum is:

$$\frac{dW}{kdk} \propto (1 - \frac{2k}{M})^{N-3}. \tag{4}$$

In this paper we consider only the quark fusion mechanism of hadronization, as the quark exchange mechanism can take place only in nuclear matter when a quasmon has neighboring nucleons. To dissociate a quasmon into a residual quasmon and an outgoing hadron one needs to calculate the probability for two quark-partons to have the effective mass of the outgoing hadron. To do that it is necessary to calculate the spectrum of the second quark-parton. The secondary spectrum can be calculated in the same way as the primary spectrum (4) where N should be substituted by $N-1$. Using (3) the spectrum can be written in the form

$$\frac{dW}{qdq} \propto \left(1 - \frac{2q}{M\sqrt{1-\frac{2k}{M}}}\right)^{N-4}, \tag{5}$$

where q is the energy of the second quark-parton in the CMS of $N-1$ quark-partons.

An additional equation comes from the mass shell condition for the outgoing hadron:

$$\mu^2 = 2\frac{k}{\sqrt{1-\frac{2k}{M}}}\cdot q\cdot(1-\cos\theta), \tag{6}$$

where μ is a mass of the outgoing hadron, and θ is the angle between the momentum directions of the two quark-partons in the CMS of $N-1$ quarks. Now the kinematical quark fusion probability can be calculated for any primary quark-parton with energy k as an integral:

$$P(k,M,\mu) = \int \left(1 - \frac{2q}{M\sqrt{1-\frac{2k}{M}}}\right)^{N-4} \cdot \times\, \delta\left(\mu^2 - \frac{2kq(1-\cos\theta)}{\sqrt{1-\frac{2k}{M}}}\right) qdqd\cos\theta. \tag{7}$$

Using the δ-function[1] to perform the integration over q one gets:

$$P(k,M,\mu) = \int \left(1 - \frac{\mu^2}{Mk(1-\cos\theta)}\right)^{N-4}$$

[1] If $g(x_0)$=0, $\int f(x)\delta\left[g(x)\right]dx = \int \frac{f(x)\delta[g(x)]}{g'(x)}dg(x) = \frac{f(x_0)}{g'(x_0)}$

$$\times \left(\frac{\mu^2 \sqrt{1-\frac{2k}{M}}}{2k(1-\cos\theta)} \right)^2 d\left(\frac{1-\cos\theta}{\mu^2} \right), \quad (8)$$

or

$$P(k,M,\mu) = \frac{M-2k}{4k} \int \left(1 - \frac{\mu^2}{Mk(1-\cos\theta)} \right)^{N-4} \times d\left(1 - \frac{\mu^2}{Mk(1-\cos\theta)} \right). \quad (9)$$

The maximum value of $z = 1 - \frac{2q}{M_{N-1}} = 1 - \frac{\mu^2}{Mk(1-\cos\theta)}$ is

$$z_{\max} = 1 - \frac{\mu^2}{2Mk}, \quad (10)$$

when $\cos\theta = -1$, and the minimum value of z is 0 when $\cos\theta = 1 - \frac{\mu^2}{M\cdot k}$. So the range of θ is: $-1 < \cos\theta < 1 - \frac{\mu^2}{M\cdot k}$. The integrated kinematical quark fusion probability (in the range from 0 to z) is

$$\frac{M-2k}{4k\cdot(N-3)} \cdot z^{N-3} \quad (11)$$

and the total kinematical probability of hadronization of the quark-parton with energy k to a hadron with mass μ is

$$\frac{M-2k}{4k\cdot(N-3)} \cdot z_{\max}^{N-3}. \quad (12)$$

Equations (11) and (12) can be used for randomization of z:

$$z = \sqrt[N-3]{R} \cdot z_{\max}, \quad (13)$$

where R is a random number uniformly distributed in the interval (0,1).

The equation (12) can be used to create a competition between different hadrons in the hadronization process. In calculating relative probabilities for different hadrons one can use only a $z_{\max}^{N-3}$ term in (12) as the rest is a constant for all candidates for the outgoing hadron, but in addition it is necessary to take into account the quark content of a candidate h and its spin s_h. As a result the relative probability can be calculated as

$$P_h(k,M,\mu) = (2s_h+1) \cdot z_{\max}^{N-3} \cdot C_Q^h, \quad (14)$$

where C_Q^h is the number of combinations of the quark content of the particular candidate from the quark content of the quasmon. We used the following quark wave functions for η and η' mesons: $\eta = \frac{\bar{u}u + \bar{d}d}{2} - \frac{\bar{s}s}{\sqrt{2}}$, $\eta' = \frac{\bar{u}u + \bar{d}d}{2} + \frac{\bar{s}s}{\sqrt{2}}$. No mixing was assumed for the ω and ϕ meson states: $\omega = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}}$, $\varphi = \bar{s}s$. Using these relative probabilities one can randomly generate the type of the outgoing hadron.

There is one more model restriction to the hadronization process. We assume that the quark content of the residual quasmon must have the quark content of either one or two real hadrons. When quantum numbers of a quasmon, determined by its quark content, cannot be represented by quantum numbers of a real hadron, the quasmon is considered to be a virtual hadron molecule such as $\pi^+\pi^+$ or $K^+\pi^+$, in which case it is defined in the CHIPS model as the Chipolino pseudo-particle.

To fuse quark-partons and create the decay of the quasmon into the selected hadron and a residual quasmon, one needs to randomly generate the residual quasmon mass m, which in fact is the mass of the residual $N-2$ quarks. Using an equation similar to (3) one finds that

$$m^2 = z \cdot (M^2 - 2kM). \quad (15)$$

Now using equation (13), the mass value of the residual quasmon can be randomized as

$$m^2 = (M-2k)\cdot(M - \frac{\mu^2}{2k}) \cdot \sqrt[N-3]{R}. \quad (16)$$

Thus, the final state hadron at this step is generated as a product of the two-particle decay of the initial quasmon into the hadron and the residual quasmon with the generated mass value.

This iterative hadronization process continues while the residual quasmon mass remains greater than $m_{\min}$, where the definition of $m_{\min}$ depends on the type of quasmon (hadron-type, or Chipolino-type). For the hadron-type residual quasmon

$$m_{\min} = m_{\min}^{QC} + m_{\pi^0}, \quad (17)$$

where $m_{\min}^{QC}$ is the minimum hadron mass for the residual quark content (QC). For the Chipolino-type residual quasmon consisting of hadrons h_1 and h_2

$$m_{\min} = m_{h_1} + m_{h_2}. \quad (18)$$

These conditions assure that the quasmon in the iterative process has always enough energy to decay into at least two final state hadrons, conserving four-momentum and charge.

If the remaining CMS energy of the residual quasmon falls below $m_{\min}$, then the hadronization process finishes by a final two-particle decay. If the parent quasmon is a Chipolino consisting of hadrons h_1 and h_2, then a binary decay of the parent quasmon into m_{h_1} and m_{h_2} takes place. If the parent quasmon is not a Chipolino then the decay into $m_{\min}^{QC}$ and m_h takes place. The decay into $m_{\min}^{QC}$ and m_π^0 is always possible in this case because of condition (17).

If the residual quasmon is not Chipolino-type, and $m > m_{\min}$, the hadronization loop can still be finished by the resonance production mechanism, modeled following the concept of parton - hadron duality [12]. If the residual quasmon mass value m is in the vicinity of some resonance with the same quark content (e.g. ρ or K^*), there is a probability for the residual quasmon to convert to this resonance.[2] In the present version of the CHIPS event generator the probability to convert to the resonance is calculated as

$$P_{\text{res}} = \frac{m_{\min}^2}{m^2}. \quad (19)$$

[2] For the comparison of the quark contents, the quark content of the quasmon is reduced by canceling of the quark-antiquark pairs of the same flavor.

With this probability the resonance with the squared mass value m_r^2 closest to m^2 is selected, and the binary decay of the quasmon into m_h and m_r takes place.

In the future when experimental data will be more detailed one can take into account angular momentum conservation, and also C-, P- and G-parity conservation. In the present version of the CHIPS event generator, η and η' are suppressed by a factor 0.3. The factor was tuned for the experiments on antiproton annihilation at rest in liquid hydrogen and can be different for other hadronic reactions. One can vary it when describing other reactions.

In addition to this parameter we had two more parameters. One of them is the suppression of heavy quark production (the so-called s/u parameter) [13]. For the proton-antiproton annihilation at rest the strange quark-antiquark sea was found to be suppressed by the factor $s/u = 0.1$. It is less than $s/u = 0.3$, which is used as a default in the JETSET [13] event generator. It can be because quarks and anti-quarks of colliding hadrons are forming an initial non-strange sea, and the strange sea is suppressed by the OZI rule [15]. This is a complicated question which is still being discussed by theorists [16] and demands further experimental measurements. The s/u parameter can be different for other reactions. In particular, for the e^+e^- reactions it can be closer to 0.3.

The temperature parameter T has been fixed at $T =$ 180 MeV. In earlier versions of the model we have found that using such a value we could successfully reproduce spectra of outgoing hadrons in different types of medium-energy reactions.

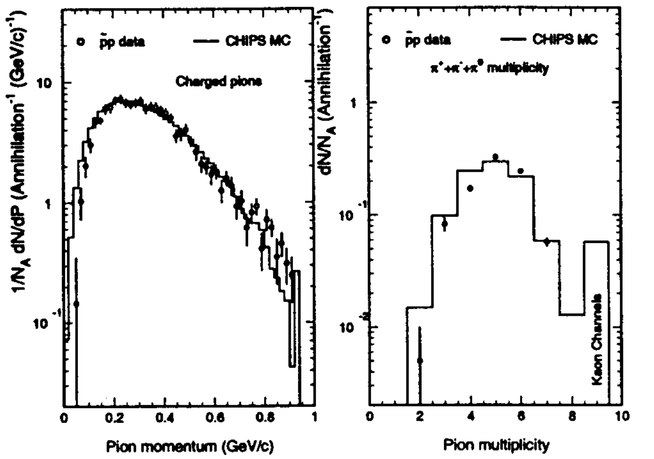

Fig. 1. Left figure (a): momentum distribution of charged pions produced in proton-antiproton annihilation at rest. Points with errors: experiment [18], histogram: CHIPS MC. The experimental spectrum is normalized to the measured average charged pion multiplicity 3.0. Right figure (b): pion multiplicity distribution. Points are taken from compilations of experimental data [17], histogram: CHIPS MC. The number of events with kaons in the final state is shown in the bin corresponding to the pion multiplicity 9, where no real 9-pion events are generated or observed experimentally. The percentage of annihilation events with kaons in the model is close to the cited experimental value of 6% [17].

3 Comparison with data

We used these parameters to fit not only the spectrum of pions (Fig. 1,a) and the multiplicity distribution for pions (Fig. 1,b) but also branching ratios of various measured [17,18] exclusive channels (Figs. 2, 3, 4). In Fig. 4 one can see many decay channels with higher meson resonances. The relative contribution of the events with meson resonances produced in the final state is 30 - 40 percent, roughly in agreement with experiment. The agreement between the model and experiment for particular decay modes is within a factor of 2-3 except for the branching ratios to higher resonances, for which it is not completely clear what is the definition of a resonance in a concrete experiment. In particular, for the $a_2\omega$ channel the mass sum of final hadrons is 2100 MeV with a full width of about 110 MeV while the total initial energy of the p$\bar{\text{p}}$ annihilation reaction is only 1876.5 MeV. This decay channel can be formally simulated by an event generator using the tail of the Breit-Wigner distribution for the a_2 resonance, but it is difficult to imagine how the a_2 resonance can be identified 2Γ away from its mean mass value experimentally.

To model fragmentation into baryons the POPCORN idea [19], which assumes the existence of diquark-partons, was used. This assumption of massless diquarks is somewhat inconsistent at low energies, as is the assumption of massless s-quarks, but it is simple and it helps to generate baryons the same way as mesons.

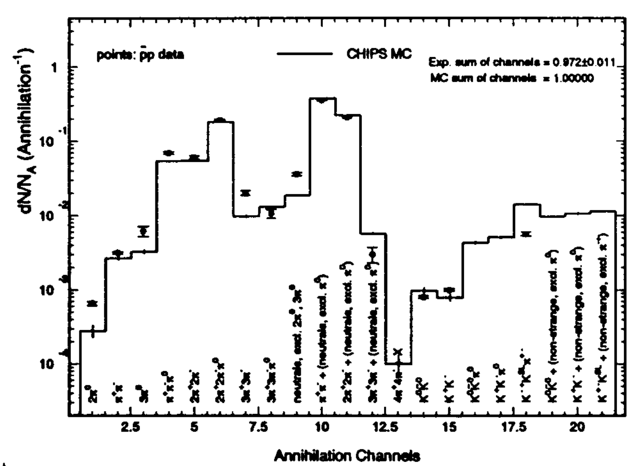

Fig. 2. Branching probabilities for different channels in proton-antiproton annihilation at rest. Points with errors: experiment [17], histogram: CHIPS MC.

Baryons are heavy, and the baryon production in p$\bar{\text{p}}$ annihilation reactions at medium energies is very sensitive to the value of temperature. If the temperature is low, the baryon yield is small, and the mean multiplicity of pions increases very noticeably with CMS energy (Fig. 5). For higher temperature values the baryon yield reduces the pion multiplicity at higher energies. The existing experi-

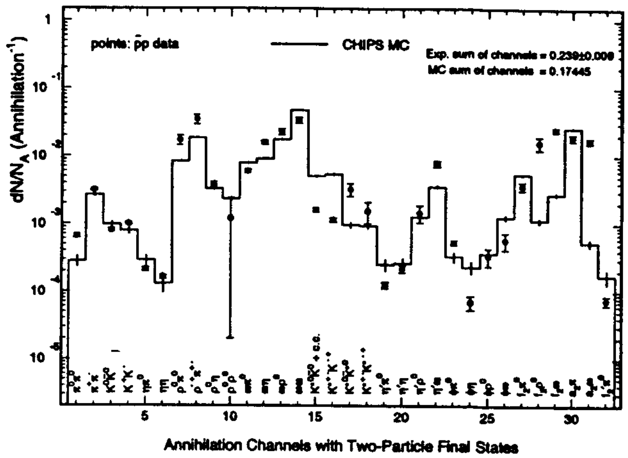

Fig. 3. Branching probabilities for different channels with three-particle final states in proton-antiproton annihilation at rest. Points with errors: experiment [17], histogram: CHIPS MC.

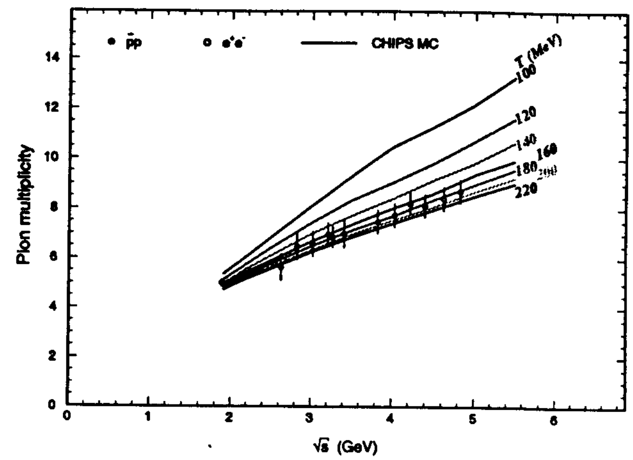

Fig. 5. Average meson multiplicities in proton-antiproton and in electron-positron annihilation, as a function of the CMS energy of the interacting hadronic system. Points with errors: experiment [20], lines: CHIPS MC calculation at different values of the critical temperature parameter T.

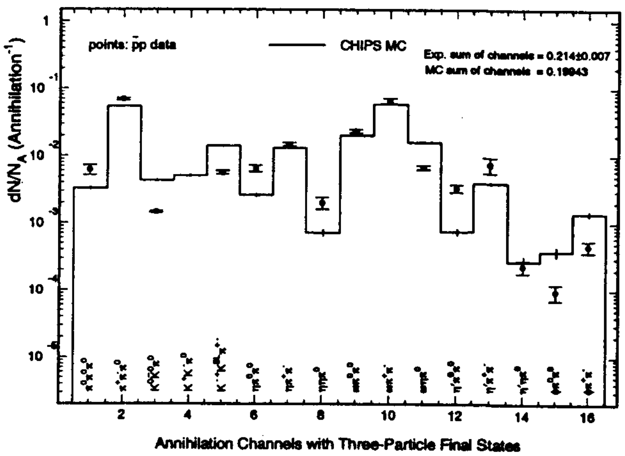

Fig. 4. Branching probabilities for different channels with two-particle final states in proton-antiproton annihilation at rest. Points with errors: experiment [17], histogram: CHIPS MC.

mental data [20] (Fig. 5) can be considered as a kind of "thermometer" for the model. This "thermometer" confirms the critical temperature value of about 200 MeV.

4 GEANT4 implementation

The kernel of the CHIPS event generator is written in C++ within the framework of GEANT4 and at this stage produces a vector of hadrons as an output. The events are not weighted. One can use weights to take into account the interference of different fragmentation amplitudes using intermediate (before decay) resonance channels which are present in the output vector of hadrons. The implementation is still preliminary, and designs will be presented in subsequent publications.

The basic architectural concept is that of a kernel-adapter-framework pattern. In this pattern, software is contained in well-defined categories, and interfaces to the application framework are made in the form of adapters. The CHIPS category in GEANT4 hence consists of a set of kernel classes, a set of utility classes, and a set of interface classes that allow the use of the CHIPS kernel in numerous conditions for shower simulation and final state generation. Particle classes and nuclear properties are re-used as far as applicable. For the present study, a preliminary implementation of the kernel was done and tested, and the interface of the GEANT4 framework for hadronic shower simulation was implemented for nucleon-antinucleon annihilation.

5 Conclusion

The CHIPS model presented is an attempt to use the set of simple rules governing microscopic quark-level behavior to model macroscopic hadronic systems with a large number of degrees of freedom. Apart from the natural rules of conservation of quantum numbers and relativistic kinematics, the hypothesis of *quark fusion* and the hypothesis of a *critical temperature* are implemented.

The *quark fusion* hypothesis determines the rules of production of the final state hadrons, with energy spectra reflecting the momentum distribution of quarks in the system.

Qualitatively, the hypothesis of a *critical temperature* assumes that the quark-gluon hadronic system (quasmon) cannot be heated above a certain temperature. Adding more energy to the hadronic system increases the number of constituent quark-partons, keeping the temperature constant. The critical temperature $T = 180 - 200$ MeV is the principal parameter of the model.

In this paper we have shown that such an approach, in combination with reasonable model assumptions on the mechanism of final state quasmon conversion into hadron resonances, may be used successfully to model hadron multiplicities and spectra in multifragmentation processes of nucleon-antinucleon annihilation.

The essential part of the model is that it can be only implemented in full in the form of a computer code – Monte Carlo event generator. We have found that the programming environment created in the new GEANT4 high energy physics simulation package is suited ideally for the development and use of the code. It can be used as a tool for the Monte Carlo simulation of a wide spectrum of hadronic reactions. The CHIPS event generator can be used not only for the "phase-space background" calculations in place of the standard GENBOD routine [14] but even for taking into account the reflection of resonances in connected final hadron combinations. Thus it can be useful for the physics analysis too, while the main range of its application is the simulation of the evolution of hadronic and electromagnetic showers in matter at medium energies.

We see an open field for the further development of the model. Same basic ideas can be used to model multifragmentation of nuclei at medium energies in hadron-nuclear or photonuclear reactions; we plan to continue the series with the implementation of nuclear fragmentation mechanisms into the model.

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